EXHIBIT H

E & P NOTES

Swanson's 30-40-30 rule

A. Hurst, G. C. Brown, and R. I. Swanson

ABSTRACT

Evaluation of the possible range of reserves associated with a prospect is a key part of risk taking in hydrocarbon exploration. The challenge of presenting a range of geologically possible models for a range of prospect reserve estimates is addressed using Swanson's 30-40-30 rule. Swanson's rule defines the mean as $0.3P_{10} + 0.4P_{50} + 0.3P_{90}$, and provides a good approximation to the mean values for modestly skewed distributions. Pragmatic and mathematical justifications for this rule are given. Applications of the rule to a historical field size distribution and a specific prospect evaluation demonstrate its efficacy in handling routine problems in hydrocarbon exploration, with particular reference to use with the lognormal distribution.

INTRODUCTION

Translation of geological concepts and models used in exploration prospect evaluation into a language that is amenable to economic evaluation is a challenging task (Rose, 1987). The uncertainty associated with the geological parameters used to calculate resources or reserves is difficult (if not impossible) to quantify. A traditional approach to estimation of the most likely reserves in a prospect is to assume that the midpoint (median) values for areal extent, pay thickness, and recovery factor can be estimated and their triple product calculated. Monte Carlo type or other probabilistic simulation procedures can then be applied to generate ranges of reserve estimates from these individual parameters. This approach makes the assumption that both larger and smaller cases for each required value (leading to a high reserves case and a low reserves case, respectively) are possible to estimate and geologically reasonable.

The triple product approach may be argued to be too simplistic in some applications. For example, in situations where there is a high level of confidence in the mapped geometry of a prospect and, possibly, where a direct hydrocarbon indicator is present, one may argue that it unnecessarily oversimplifies reality. Although we

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AUTHORS

A. Hurst ~ *Department of Geology and Petroleum Geology, University of Aberdeen, Aberdeen, AB24 3UE, United Kingdom; a.hurst@geol.abdn.ac.uk*

Andrew Hurst received a Ph.D. in geology from the University of Reading in 1980. From 1981 until 1990, he worked for Statoil, Norway, in the petroleum technology group. From 1990 to 1992, he was employed as an exploration geologist by Unocal UK. In 1992, he assumed the new Shell Chair of Production Geoscience at Aberdeen University. His main research interest is in reservoir characterization, particularly of turbidite reservoirs. He is the chief editor of *Petroleum Geoscience.*

G. C. Brown ~ *Department of Mathematical Sciences, University of Aberdeen, Aberdeen, AB24 3UE, United Kingdom. Current address: Office for National Statistics, 1 Drummond Gate, London, SW1V 2QQ, United Kingdom; Gary.Brown@ons.gov.uk*

Gary Brown received a B.Sc. degree (1989) in mathematics and a Ph.D. (1995) in statistics from the University of Birmingham. From 1996 to 1999 he was employed as a research fellow in statistics at the University of Aberdeen, where he also ran an internal and external statistical consultancy service. His main research interest is in multivariate regression, especially in the associated diagnostics. Since 1999 he has been employed as a methodologist in the Office for National Statistics.

R. I. Swanson ~ *deceased early 2000.*

Roy Swanson received a B.S. degree (1953) in geology from the University of Pittsburgh and an M.S. degree (1955) in geology from Michigan State University. He worked for Exxon Corporation first as a geophysicist in Wyoming, Colorado, and Venezuela and then as a geologist in New Orleans. In 1969, he entered the area of exploration economics and in 1981 became coordinator of Inland Exploration analysis for Exxon. He retired from Exxon in 1992.

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Our thanks to Roy Swanson for his invaluable contributions to this paper, and to the field of petroleum geology in general. It was with great sadness we heard of his passing away in March 2000.

believe that these considerations may hold true for exploration in mature hydrocarbon plays, experience tends to demonstrate the frailty of our predrill knowledge and a tendency toward overconfidence. Assumption of an unrealistic slablike (area $\times$ thickness) geometry may fail to capture a realistic three-dimensional prospect geometry but does allow simple generation of average values. The heterogeneity of reservoirs, both depositional and diagenetic, actually produces a range of recovery factors, even within a restricted area of a play, which is commonly confirmed by production history. However, adopting a more sophisticated approach to reserve estimation where technical risk cannot be adequately constrained may only serve to introduce uncertainty in estimates rather than constrain them.

If prospect sizes lie on a lognormal curve, taking a midpoint to represent the most likely reserves case does not yield the true median. Equally significant is that, during the process of prospect generation, geologists and managers commonly have difficulty in envisaging prospects smaller than the 90th percentile or larger than the 10th percentile. To take account of this, Swanson, in an Exxon internal memorandum (1972, personal communication), encouraged the use of average prospect size rather than a subjective most likely case. To achieve this, he proposed use of a rule when explaining prospect economics. This rule, called the "30-40-30" rule, is commonly referred to as "Swanson's rule" and has since been widely used in the hydrocarbon exploration industry (Megill, 1984; Rose, 1991; Rose and Thompson, 1992).

THE 30-40-30 RULE

The rule set out to select three sizes along the reserves distribution curve, having the aim of reflecting the range of the distribution curve and, where weighted by discrete probabilities, giving the average value of the prospect reserves.

The three sizes were proposed as

- the medium (50th percentile $\equiv$ median, P_{50}), which half the reserves were larger than, half smaller;
- the maximum (10th percentile, P_{90}), which only 10% were larger than;
- the minimum (90th percentile, P_{10}), which 90% were larger than.

Probabilities were then assigned to P_{10}, P_{50}, and P_{90} based on the proportions of the total range 0–100% appropriate to each. For P_{50}, the ranges to P_{10} and P_{90} were 0.40 of the total range in each case (i.e., 10–50% = 0.40 of the total range). These 0.40 ranges were then halved and one half assigned to each (i.e., 50–70% = 0.20 of total range assigned to P_{50} and 70–90% = 0.20 of total range assigned to P_{90}). By this method the probability of P_{50} was 0.20 + 0.20 = 0.40 (i.e., 30–50% and 50–70%). Following this,

each of P_{10} and P_{90} already had probabilities of 0.20 assigned to them. Added to these probabilities were 0.10 to represent the tails of the distribution beyond (i.e., 90–100% = 0.10 of total range for P_{90}), which gave probabilities of 0.30 for each of P_{10} and P_{90} (i.e., 70–90% and 90–100% for P_{90}). The weightings of P_{10}, P_{50}, and P_{90} were thus 0.30, 0.40, and 0.30, respectively, summing to a value of 1 (i.e., total range 0–100%) as required for a probability distribution. This defines Swanson's 30-40-30 rule, which was noted to work for all except highly skewed distributions.

JUSTIFICATION

In the original memorandum, Swanson noted that having different choices of percentiles for the maximum and minimum sizes (and corresponding new probabilities) did not return as accurate an estimate of the average as the 30-40-30 rule. Thus no equivalent 20-60-20 rule would be as accurate.

A test for the variate being lognormal was originally proposed as

$$\frac{\text{Minimum Value}}{\text{Most Likely Value}} = \frac{\text{Most Likely Value}}{\text{Maximum Value}} \quad (1)$$

If values satisfied equation 1 then this was taken to indicate a lognormal distribution. Equation 1 was also given as a method of finding an unknown value if lognormality was already assumed and only two values were known. The connection between log symmetry and lognormality in equation 1 is not mathematically justifiable, and consequently the test was later dropped. However, the lognormality of the reserves was exemplified by the reserves of known contemporary fields plotting a straight line on lognormal probability paper (Figure 1). The precise percentiles and their corresponding probability weightings, chosen to estimate the mean of such a positive skewed distribution as the lognormal, were actually found empirically by computer simulation.

A mathematical justification of the 30-40-30 rule is provided in the Appendix.

APPLICATIONS

During prospect evaluation, the main application of Swanson's rule is to translate resource assessment values to economic values. Further uses are in the plotting of possible reserves for individual prospects and comparison of prospect size with historical field size distributions.

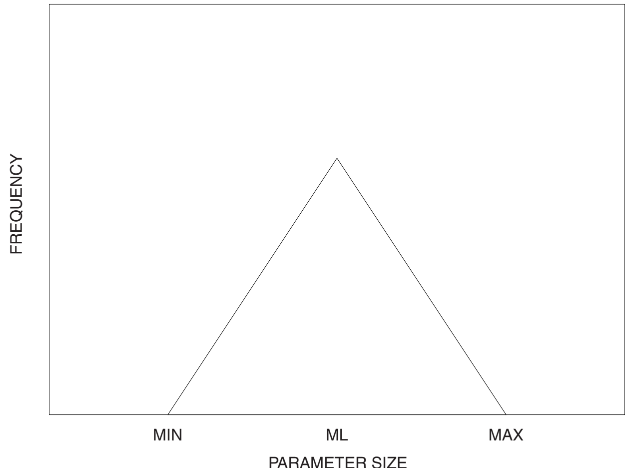

Figure 1. Symmetric triangular distribution. MIN = minimum; ML = most likely; MAX = maximum.

Whether Swanson's rule, or a more sophisticated approach, is used for reserve estimation is determined by user confidence in the various risk parameters being assessed. High confidence follows from experience and analysis of all appropriate data using state-of-the-art technology. First-time participation in a play, or in the testing of a new play, is unlikely to be accompanied by such confidence, and the triple product approach will be more attractive. Although modern computational power makes it relatively fast and straightforward to generate many resource and economic scenarios, defining the parameters necessary to generate resource estimates remains time intensive and subjective.

Prospect Reserve Estimation

When estimating the likely range of reserves for a prospect, an exploration geologist is always faced with problems associated with the limitations of the data from which the prospect has been generated. A common misconception in estimating reserves is that the minimum and maximum values for specific parameters (e.g., closure, thickness, porosity, recovery, etc.) plot on frequency diagrams having a value of zero (Figure 1). This is a statistical misconception because, by plotting at zero, the high and low values cannot occur. Further, this means that any estimates lower or higher than these values also cannot exist.

Therefore, to accommodate the need to include a range of values greater than the known values, reserves are plotted as percentiles. It is important to realize that all we know is not all there is to know and that normal practice in subsurface studies involves dealing with incomplete data (Foley et al., 1997).

Additional to the misconception regarding the meaning of statistical distributions is a tendency for exploration geologists to believe that a prospect map, commonly the product of a highly individual exercise, is the only (certainly, most likely) map for a prospect (Megill, 1984). Although it only takes a little considered thought or practical experience to dismiss this belief, human nature tends to anchor estimates to original concepts and interpretations (Rose, 1987). Capen et al., (1971) and Bain (1993) give examples that demonstrate how different exploration geologists, using similar data, can produce a range of different assessments of hydrocarbon exploration prospectivity. In the light of this tendency to anchor onto the reserves case perceived as most popular by the exploration geologist involved, the generation of highside and lowside reserve cases is always challenging. Once generated, though, use of lognormal probability plots allows simple extrapolation above and below an exploration geologist's favored reserves case (Megill, 1984; Rose, 1987; Rose and Thompson, 1992).

Unfortunately, no rules exist for constructing prospect lognormal probability plots. Equally, no rule exists for use of the probability axis. Therefore, P_{90} is quoted by different users to represent a low probability of occurrence (there is a 10% probability of a reserve being greater than or equal to the P_{90} case) or a high probability of occurrence (there is a 90% probability of a reserve being greater than or equal to the P_{90} case). Here we adopt the former use (Rose, 1987; Rose and Thompson, 1992). In constructing a lognormal probability curve, Megill (1984) favors defining the P_{50} and P_{10} (lowside estimate) values and extrapolating to the P_{90} (highside). The exploration geologist's favored reserves case, perhaps termed the most likely technical case, is a reasonable place to start.

Intuitively, one may expect the most likely case to be close to the P_{50} value on a probability curve, though this need not be the case. However, by plotting the most likely case as P_{50} one can begin to generate a plot for a reserve distribution if a value for P_{10} can be defined (Rose, 1991). A pragmatic approach to defining P_{10} is to assume that it corresponds to the economic cutoff for a single well discovery that when converted to a producing well would be expected to recoup all costs. Clearly, this definition of P_{10} varies enormously, depending on local conditions and operator economics. Once P_{10} and P_{50} are plotted, a P_{90} value may be extrapolated. At this point, it is wise to consider how reasonable the extrapolated P_{90} and P_{99} reserve estimates are, both by using input parameters specific to the prospect and by comparison with historical field size distributions for the trend or play, if available. If the P_{90} and P_{99} estimates seem unreasonably large given the geological constraints (that must be geologically feasible), the most likely case should be adjusted to a higher value (toward P_{60}) to increase the gradient of the lognormal probability curve (Figure 2, line a). If the P_{90} estimate appears too small, because it is easier to make a substantially larger reserve estimate using reasonable geological input parameters, the most likely case should be adjusted to a lower probability (toward P_{40}, Figure 2, line b).

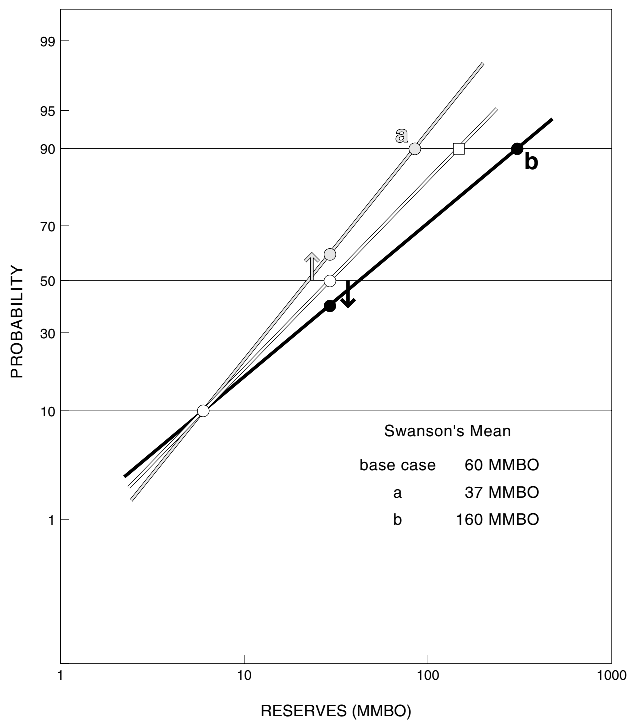

Figure 2. Adjusting lognormal curves for estimation of prospect size. In the initial plot (base case) P_{10} is assigned as an economic cutoff value, P_{50} is the exploration geologists' most likely case, and Swanson's mean values, using the 30-40-30 rule, are shown. P_{90} is extrapolated from these values. Line a illustrates a scenario where the original most likely case is considered too high and lowered to less than P_{50}. Line b illustrates a scenario where the original most likely case is considered too low and moved to greater than P_{50}. MMBO = million bbl of oil.

This procedure should never be treated merely as a numbers game. Any changes to the reserves log(probability) must be as a consequence of geologically reasonable adjustments to mappable parameters. Good practice involves generating several reserve distribution curves for a prospect until one feels comfortable that the distribution is representative of the interpreter's perception of reality.

A common error in prospect evaluation is to assume that a lognormal reserves distribution has successfully captured the entire spread of reasonable reserves for a prospect. Whether the reserves distribution has been generated manually or by using software, visual checks should be made that high case reserves ($>P_{90}$) fall within the bounds of reason; prospect area and thickness do not exceed what is physically possible to accommodate; and reservoir properties do not exceed reasonable values for porosity and recovery factors.

Historical Data

Field size distributions in a basin or play commonly approximate lognormal trends (Megill, 1984; David, 1996). When prospects are generated it is commonly useful to compare reserve estimates with known reserves from the same trend. This practice aims to provide an objective comparison of prospect reserves with known reserves. We do not discuss the issue of determining the similarity of discoveries to prospects in any further detail, except to state that combining and comparing dissimilar data prejudices statistical relevance.

Plotting historical field size distributions is done in the same manner as prospect reserves. An example of historical data from the central North Sea (United Kingdom and Norway) compiled during the period 1989–1991 is used to demonstrate their use for supporting evaluation of exploration risk. Field and discovery data are summarized in Table 1, and the full historical trend is shown in Figure 3. Three prospects within the trend, X, Y, and Z, are marked on the distribution in Figure 3. The implications to be drawn from the position of the prospects are that, given the historical context of exploration, X (250 million bbl of oil [MMBO]) is relatively unlikely to occur (a 9.5% probability of a field of 250 MMBO or larger occurring, i.e., 250 MMBO would correspond to $X_{9.5}$ in Appendix equation 5); Y (45 MMBO) has a reasonable probability of occurrence (a 46% probability of a field of 45 MMBO or larger occurring, corresponding to X_{46} in equation 5); and Z (13 MMBO) is likely to occur (a 78% probability of a field of 13 MMBO or larger occurring, corresponding to X_{78} in equation 5). These probabilities of occurrence are not probabilities of success for the individual prospects. Mitigating circumstances may exist that will influence this simple interpretation of the data.

Table 1. Fields and Discoveries in the Upper Jurassic, Salt-Related Play in the United Kingdom and Norwegian Central North Sea*

Field/Well	Date of Discovery	Reserves (MMBO)
Fulmar	1975	462
Clyde	1978	150
21/24-1	1978	10
Guillemot "A"	1979	50
Gyda	1980	195
Iris	1980	10
30/12b-2	1981	29
21/19-1	1981	10
N7/12-5	1981	12
Kittiwake	1981	70
21/15a-1	1981	50
21/13b-1	1982	80
Mime	1982	15
Angus	1983	8
21/19-2	1983	16
21/20-2	1983	20
Duncan	1985	20
N15/12-5	1986	100
29/6a-3	1987	30
22/13a-1	1989	90
N2/4-14	1990	175

*Data for fields are public domain information available in the period 1989–1991. Other data are estimates obtained from independent studies and scout data. Prefix N denotes a Norwegian sector well.

CASE STUDY

Prospect Ω is a three-way dip closure into a major basin margin fault having small areas of four-way dip closure and an inferred stratigraphic pinch-out to the east that provides substantial possible upside to reserve estimates (Figure 4). Substantial oil accumulations are known downdip and to the west of prospect Ω, where the target reservoir is thick and productive. Stratigraphic pinch-out is key to providing closure but also makes implicit that presence, thickness, and quality of

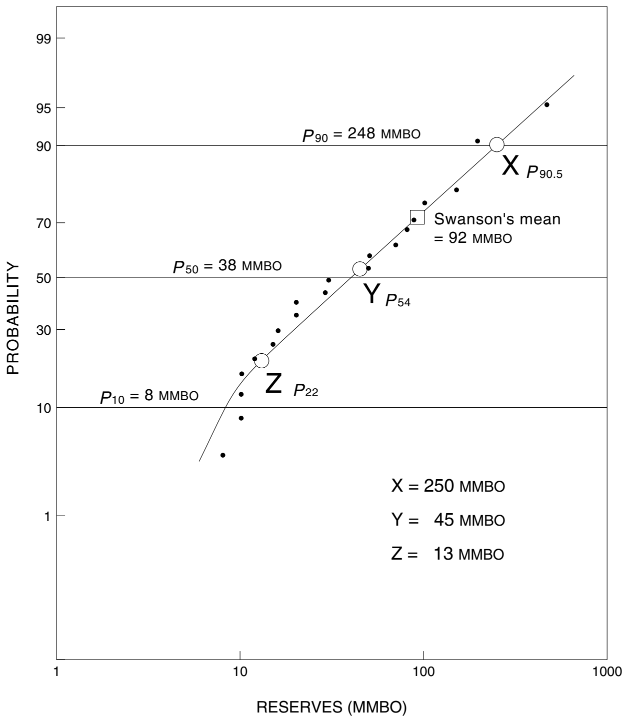

Figure 3. Field size distribution for the Upper Jurassic, salt-related play (United Kingdom and Norway, North Sea) compiled during 1989–1991. Prospects of the size of X, Y, and Z (Table 1) have reserve estimates of 250, 45, and 13 million bbl of oil (MMBO), respectively.

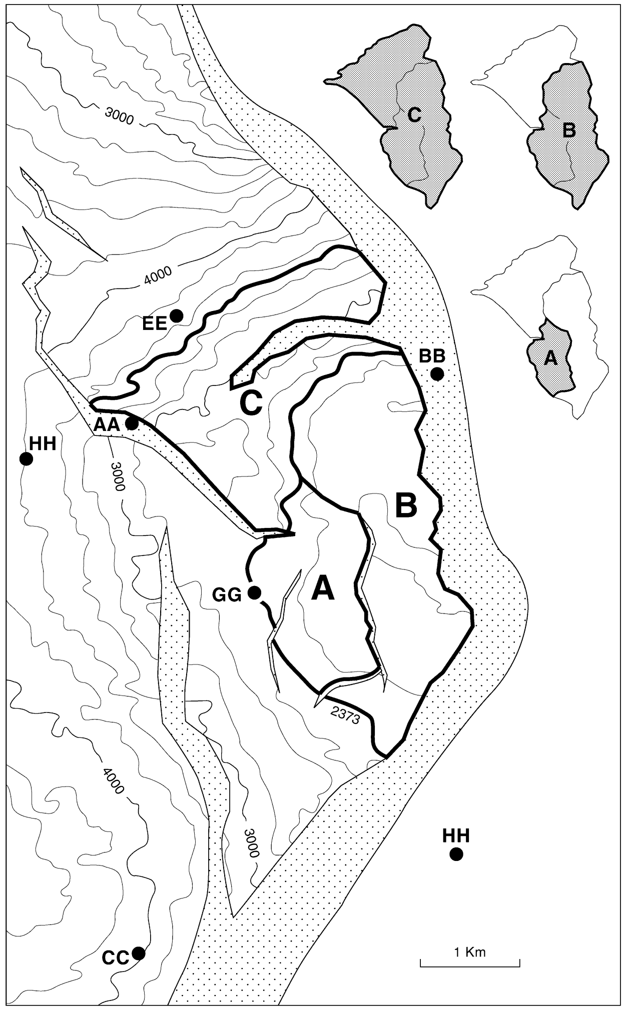

Figure 4. Prospect Ω. A three-way dip closure into a basin margin fault. Areas A, B, and C are three possible geological interpretations of hydrocarbon distribution (outline areas also shown as insets). Well names are indicated by paired letters.

the target reservoir are significant uncertainties. The reservoir is known to thicken immediately downdip and to the west, where several dry holes are located. Further east and to the south the reservoir is not present. At the time of evaluation, no similar play concept had been tested.

Constraints

As no direct hydrocarbon indicator is identifiable on the seismic data to aid mapping of hydrocarbon distribution, the approach to reserve mapping was to map closure manually, honoring well data, and to infer possible hydrocarbon distribution. Well GG penetrated an 8 m thick oil column and an oil-water contact in the target reservoir horizon; wells AA and BB failed to locate the target reservoir. All other wells located a water-bearing reservoir.

Well AA is located on the footwall of a major basin margin fault. Well BB penetrated a thick unit of the top seal that directly overlies the target reservoir in wells to the west but in BB is in faulted contact with basement. This fault is interpreted to be the basin margin fault. The general thinning of the target reservoir unit from west to east (Table 2) leads to the concept of stratigraphic trapping that combines depositional pinch-out and an erosional truncation.

For the sake of simplicity, only prospect areas are disclosed in this example. In reality, variations in net pay and recovery factor were also considered in the reserve distributions (cf. Megill, 1984; Rose, 1991).

Possible Hydrocarbon Distribution

The oil-water contact proved at 2373 m allows extrapolation of a simple reserve case limited updip by a fault to the east of well GG and downdip by the fluid contact (A, Figure 4). If the fault is considered to be non-

Table 2. Well Data Adjacent to Prospect Ω

Well	Status	Reservoir Thickness
AA	Dry hole	np*
BB	Dry hole	np
CC	Dry hole	21 m
EE	Dry hole	28 m
GG	Oil well	17 m
HH	Dry hole	36 m

*np = reservoir not present.

sealing, it is possible that the reservoir can extend farther east up to the inferred stratigraphic termination (B, Figure 4); the presence and quality of the reservoir are substantial risks in this case. Both these scenarios underfill the mapped closure.

Underfilling of closure presents many possible geological scenarios that may elucidate geological risk and make possible upgrading of reserves. The trap may have received too little charge or the top seal may have insufficient capillary pressure to hold back a thicker column (possibly because of faulting). As a thick shale unit underlies the target reservoir, bottom seal leakage is not considered to be a significant element of risk. The direction from which oil has migrated into Prospect Ω is significant in determining hydrocarbon distribution. Migration is believed to be from the northwest past well EE. Given the geometry of the inferred pinch-out to the east and south, it is possible to generate a model in which a perched oil-water contact occurs (C, Figure 4). The shallower oil-water contact in well GG and in the south of the prospect is caused by physical trapping of formation water in a segment of reservoir that is bounded by the basin margin fault penetrated by well BB (assumed to be sealing) and the stratigraphic pinch-out (Figure 4).

The three possible hydrocarbon distributions (A, B, and C) for Prospect Ω are just three of an infinite range of possible reserves cases that can be described using a lognormal-probability crossplot. Using common sense and experience one should recognize that the geological risk increases from A through to C, but how does one evaluate the most likely technical case? Following the method of Rose (1991), the reserves-probability crossplot is generated by choosing to use the minimum commercial reserve value for Prospect Ω, 7 million bbl (MMB), to represent P_{10} (a 90% probability of 7 MMB or less occurring). This assumes that discovery of 7 MMB generates an incremental net present value equal to zero (at the time of the evaluation). Three reserves-probability distributions, P50A, P50B, and P90C (Figure 5), are generated using reserve estimates of 18 MMB, 45 MMB, and 92 MMB (associated with areas A, B, and C, Figure 4). Curve P50A is the most conservative reserve distribution for Prospect Ω and curve P50B the most optimistic. One can equally interpret the conservative curve as being equivalent to a high level of confidence in the outcome and the optimistic curve as a lower level of confidence. Selection of the most appropriate reserve distribution should be made by examination of geological evidence. Any reserve scenario must be geologically possible.

Examination of P values less than P_{90} (<10% probability of occurring) are particularly useful in this context because if high reserve cases require unreasonable geological conditions to exist, they are probably unrealistic. The greater than P_{90} reserves for curves P50A and P50B are values greater than 47 MMB and 280 MMB, respectively. The 280 MMB is more than three times the reserves associated with area C (Figure 4),

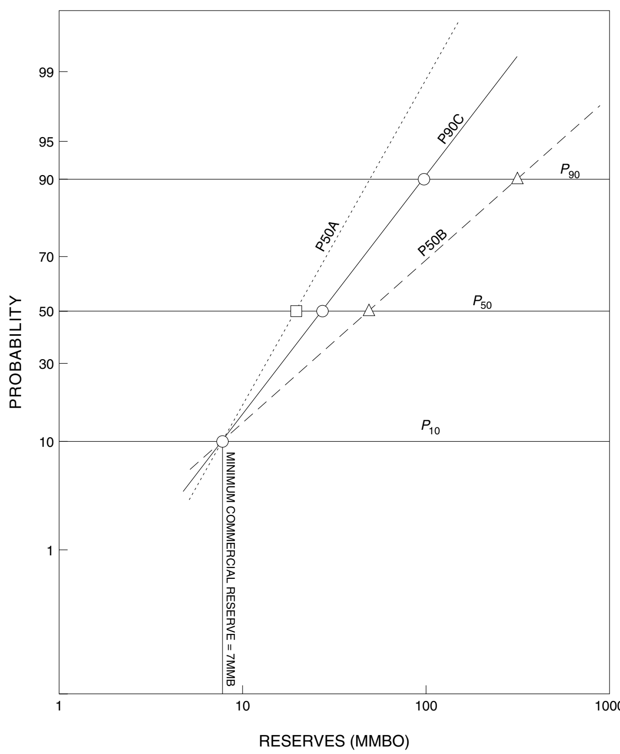

Figure 5. Reserves (log) and probability crossplot for Prospect Ω (Figure 4). All curves use 7 million bbl (MMB) as the minimum commercial reserve value. The fine dashed curve (P50A) is derived using area A as the P_{50}, the broad dashed curve (P50B) uses area B as the P_{50}, and the solid curve (P90C) uses area C as the P_{90}. MMBO = million bbl of oil.

which is an optimistic scenario for hydrocarbon distribution (perched oil-water contact). Consequently, 280 MMB seems likely to be much too large for a P_{90} value and quite possibly unlikely to occur at all. The P50B curve is rejected as representative reserves distribution for Prospect Ω because the range of reserves is impossible, or at least extremely difficult, to accommodate geologically. In contrast, curve P50A is rejected as a representative distribution because the P_{90} and P_{99} reserves (47 and 100 MMB, respectively) (Table 3) are easily accommodated geologically and the Swanson's mean (21 MMB) is small relative to the possible mapped hydrocarbon distributions (Figure 4).

As the results of further exploration of Prospect Ω are unavailable, validation of the reserves distribution is not possible. However, it seems likely that the appropriate curve lies between P50A and P50B. Geologists working Prospect Ω believed that area B (Figure 4) is quite likely to occur, despite reserves of 45 MMB being unlikely to be a reasonable P_{50} value for the prospect. Curve P90C is plotted in the belief that area C (Figure 4) is likely to be more probable as a P_{90} reserve case. Curve P90C gives a Swanson's mean of 38 MMB that seems reasonable in the context of the mapped closure and other undisclosed variations in pay and recovery factor. Equally, the P_{99} of 255 MMB is manageable as a model, having only a 1% probability of occurring.

CONCLUSION

Swanson's 30-40-30 rule is able to find the mean of modestly skewed distributions. Reserve estimates for prospects and historical field size distributions commonly approximate to modestly skewed distributions (linear trends on log-probability plots), thus the 30-40-30 rule is particularly amenable to their study. Although it may be argued that in some modern exploration scenarios the level of confidence associated with prediction of prospect geometry and hydrocarbon presence is high, practical experience of exploration drilling demonstrates a record of overconfidence in prediction. Swanson's 30-40-30 rule has a long history of application to the translation of resource assessment into economic values, however the efficacy of the method is dependent on effective representation of geologically reasonable models throughout the range of reserves represented.

In defining an appropriate curve to represent a range of reserves for a specific prospect an iterative approach is used. Typically, the P_{10} value (90% probability of finding reserves of this size or lower) can be assumed to be the minimum commercial reserves value, the most likely reserve case may be used as a provisional P_{50}, and a reserve distribution curve be generated. The validity of the curve, and hence the probability of the most likely case, can then be tested by evaluating how geologically realistic the high reserve cases are. The final reserve distribution curve is one that depicts geologically reasonable reserve cases and represents a view of confidence in their occurrence. Curve slope provides an easy assessment of the level of confidence associated with estimates.

Table 3. Reserves Data Derived from Each Reserve Distribution Curve (RD Curve) in Figure 5*

RD Curve	P_{10}	P_{50}	P_{90}	P_{99}	M_{Sw}
P50A	7	18	47	100	21
P50B	7	45	280	960	104
P90C	7	20	92	255	38

*All values are in MMB.

APPENDIX: THEORETICAL JUSTIFICATION OF SWANSON RULE

The average ($\bar{X}$) the 30-40-30 rule estimates is given by

$$\bar{X} = 0.30X_{90} + 0.40X_{50} + 0.30X_{10} \quad (2)$$

where X_α represents the point of the distribution X (i.e., distribution of reserves) that is exceeded α% of the time. The general form of equation 2, retaining symmetry but allowing the weighting and percentile parameters to vary, is

$$\bar{X}(\omega, \delta) = \omega X_{50} + \delta + (1 - 2\omega)X_{50} + \omega X_{50} - \delta \quad (3)$$

where ω is the probability and δ is the number of percentage points difference of the maximum and minimum from the medium (50th percentile, P_{50}).

The true mean of a lognormally distributed variate is known to be (Hastings and Peacock, 1975)

$$\bar{X} = m \exp\left\{\frac{1}{2}\sigma^2\right) \quad (4)$$

where $m = X_{50}$ (the median) and σ is the standard deviation of log(X). The derivation of Swanson's rule thus proceeds by calculating where the general form in equation 3 equals the true mean in equation 4. By definition

$$P(X > X_\alpha) = \frac{\alpha}{100} \quad (5)$$

where α is the percentile. Because making a transformation does not alter the relative probability

$$P(\log(X) > \log(X_\alpha)) = \frac{\alpha}{100} \quad (6)$$

The mean of the log of the lognormal variate X is log(median), and because (by definition) the log of a lognormal variate is normally distributed, then normalization can proceed. This entails transforming the variate $\log(X)$ in equation 6 into a normal variate Z having mean 0 and variance 1

$$P\left(\frac{\log(X) - \log(X_{50})}{\sigma} > \frac{\log(X_\alpha) - \log(X_{50})}{\sigma}\right) = P\left(Z > \frac{\log(X_\alpha) - \log(X_{50})}{\sigma}\right) = \frac{\alpha}{100} \quad (7)$$

Now, from tables of the normal distribution, $P(Z > 1)$, notated $\phi(1)$, is 0.1587.

For the normal distribution, probability $P(Z > 1)$ is commonly notated $\phi(1)$ and is the probability that a random selection of a variable Z, whose distribution is normal having mean 0 and variance 1, is greater than 1. For example, if the mean height of men in a town was 1.8 m with standard deviation 0.2 m (i.e., variance 0.04 m), then the variable $Z = (\text{height} - 1.8)/0.2$ would have mean 0 and variance 1, and $\phi(1)$ would be equivalent to the probability that a randomly chosen male would have height greater than 2.0 m.

Substitution of $\phi(1) = 0.1587$ into equation 7 yields

$$\sigma = \log(X_{15.87}) - \log(X_{50}) = \log\left(\frac{X_{15.87}}{X_{50}}\right) \quad (8)$$

With σ and m now known, the mean in equation 4 is completely determined, and equation 7 leads to

$$X_\alpha = X_{50} \exp\left\{\sigma\Phi^{-1}\left(\frac{\alpha}{100}\right)\right\} \quad (9)$$

where ϕ^{-1} is the inverse of ϕ, (i.e., $\phi\{\phi^{-1}(a)\} = a$ and $\phi^{-1}(0.1587) = 1$). Setting equation 3 equal to equation 4, substituting for equation 9, and solving for ω, yields

$$\omega = \frac{\exp\left\{\frac{1}{2}\sigma^2\right\} - 1}{\exp\left\{\sigma\Phi^{-1}\left(\frac{50+\delta}{100}\right)\right\} + \exp\left\{\sigma\Phi^{-1}\left(\frac{50-\delta}{100}\right)\right\} - 2} \quad (10)$$

With the choice of $\delta = 40$ (as in Swanson's rule), then $\phi^{-1}(0.1) = 1.282 = -\phi^{-1}(0.9)$. Substituting this into equation 10, because σ tends to log(1), gives a value of $\omega = 0.304$ (to 3 decimal places). This agrees closely with the 30-40-30 rule proposed by Swanson: ω increases with increasing σ, becoming 0.35 at $\sigma = \log(2.864)$.

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